SISTEMAS LINEARES I

LISTA 2

GABARITO

Q1 - MODELAGEM DE SISTEMAS LINEARES

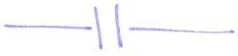
RELEMBRANDO

RESISTOR

$V = Ri$

R

CAPACITOR



C

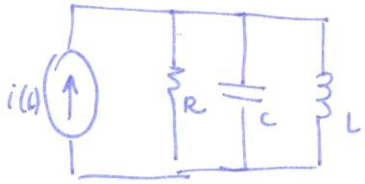
$$i_c(t) = C\frac{dV_c(t)}{dt}$$

INDUTOR

L

$$V_L(t) = L\frac{di_L}{dt}$$

(a) CIRCUITO 1



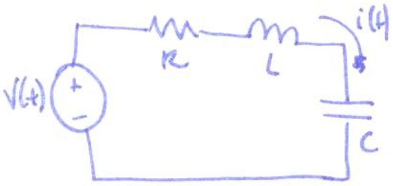
PELA LKC

$$i(t) = i_R(t) + i_c(t) + i_L(t)$$

$$\boxed{i(t) = \frac{V(t)}{R} + C\frac{dV(t)}{dt} + \frac{1}{L}\int_{-\infty}^{t} V(t)\,dt}$$

10pts

CIRCUITO 2



PELA LKT

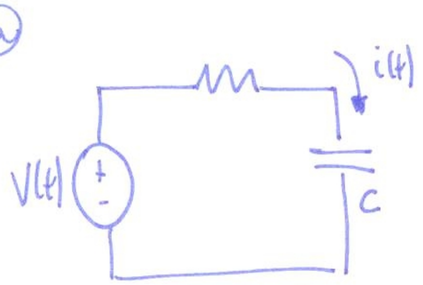
$$V(t) = V_R + V_L + V_c$$

$$\boxed{V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt}$$

10pts

Q2

a



MODELAGEM:

$$V(t) = V_R(t) + V_C(t)$$

$$V(t) = R\,i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

ESTADO ZERO → $V(t) = 0$

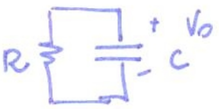
$$0 = R\,i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt \rightarrow 0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t) \rightarrow \frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0$$

SUPONDO $i(t) = e^{\lambda t}$

$$\frac{di(t)}{dt} = \lambda e^{\lambda t} \rightarrow \left(\lambda + \tfrac{1}{RC}\right)e^{\lambda t} = 0 \rightarrow \lambda + \tfrac{1}{RC} = 0 \rightarrow \boxed{\lambda = -\tfrac{1}{RC}}$$

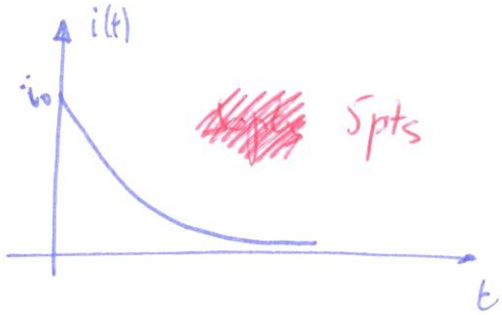
1 - COM $R = 1$, $C = 1$, e $V_0 = 1$, TEMOS:

$\boxed{\lambda = -1} \rightarrow i(t) = i_0 e^{-t}$



$V_0 = 1$

$\boxed{i_0 = 1\,A}$



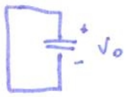
5pts

2 - COM $R \rightarrow 0$, $C = 1$, $V_0 = 2$

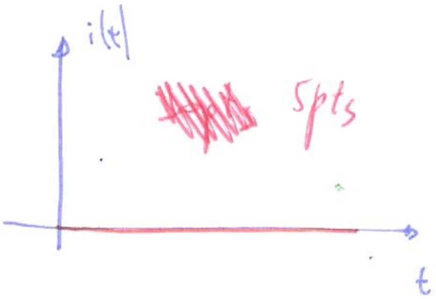
$\boxed{\lambda = -\infty}$

$i(t) = i_0 e^{-\infty}$

$i(t) = 0$



SEMELHANTE A UM CURTO CIRCUITO



5pts

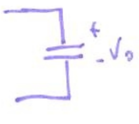
A INTERPRETAÇÃO QUE PODE SER FEITA DESSE CIRCUITO É QUE O CAPACITOR SE DESCARREGA INSTANTANEAMENTE. COM UM IMPULSO DE CORRENTE

3 - COM $R \rightarrow +\infty$, $C = 1$, $V_0 = 2$

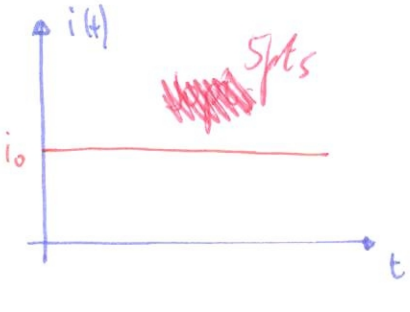
$\boxed{\lambda = 0}$

$i(t) = i_0 e^{-0}$

$i(t) = i_0$



SEMELHANTE A UM CIRCUITO ABERTO



5pts

A INTERPRETAÇÃO QUE PODE SER FEITA DESSE CIRCUITO É QUE i_0 É MUITO PEQUENA OU TENDE A ZERO POIS $R \rightarrow +\infty$ É SEMELHANTE A UM CIRCUITO ABERTO

Q2

b

MODELAGEM

$$\frac{\partial^2 i(t)}{\partial t^2} + \frac{R}{L}\frac{\partial i(t)}{\partial t} + \frac{1}{LC} i(t) = 0$$

1- $R = 3$, $C = 2$, $L = 1$, $V_c(0^-) = 1$, $I_L(0^-) = 1$

$$\frac{\partial^2 i(t)}{\partial t^2} + 3\frac{\partial i(t)}{\partial t} + \frac{1}{2} i(t) = 0 \longrightarrow \lambda_1 = \frac{-3 + \sqrt{7}}{2} \quad \lambda_2 = \frac{-3 - \sqrt{7}}{2}$$

$$i(t) = c_1 e^{\left(\frac{-3+\sqrt{7}}{2}\right)t} + c_2 e^{\left(\frac{-3-\sqrt{7}}{2}\right)t}$$

$$i(0) = i_L(0) = 1 = c_1 e^0 + c_2 e^0 = c_1 + c_2$$

P/ LKT

$$V(0) = i(0) \cdot R + V_c(0) + L\frac{di(t)}{dt} \Rightarrow 0 = i(0) \cdot R + V_c(0) + L\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} = -4$$

PARA A DERIVADA DE $i(t)$ $\rightarrow$ $i'(0) = c_1\left(\frac{-3+\sqrt{7}}{2}\right) + c_2\left(\frac{-3-\sqrt{7}}{2}\right)$

ENTÃO:

$$c_1 + c_2 = 1$$

$$c_1\left(\frac{-3+\sqrt{7}}{2}\right) + c_2\left(\frac{-3-\sqrt{7}}{2}\right) = -4 \rightarrow \begin{aligned} c_1 &= \frac{1}{2} - \frac{5\sqrt{7}}{14} \\ c_2 &= \frac{1}{2} + \frac{5\sqrt{7}}{14} \end{aligned}$$

$$i(t) = \left(\frac{1}{2} + \frac{5\sqrt{7}}{14}\right)e^{\left(\frac{-3+\sqrt{7}}{2}\right)t} + \left(\frac{1}{2} - \frac{5\sqrt{7}}{14}\right)e^{\frac{-3-\sqrt{7}}{2}t}$$

5pts

Q2 (b)

(2) $R = 2,\ C = 2,\ L = 1,\ V_c(0^-) = 1,\ I_L(0^-) = 1$

$$\frac{\partial^2 i(t)}{\partial t^2} + 2\frac{\partial i(t)}{\partial t} + \frac{1}{2} i(t) = 0 \quad \lambda_1 = \frac{4+\sqrt{2}}{2} \quad \lambda_2 = \frac{4-\sqrt{2}}{2}$$

$$i(t) = c_1 e^{\left(\frac{4+\sqrt{2}}{2}\right)t} + c_2 e^{\left(\frac{4+\sqrt{2}}{2}\right)t}$$

$$c_1 + c_2 = 1$$

$$\frac{di(t)}{dt} = -3$$

$$V(0) = i(0)R + V_c(0^-) + L\frac{di(t)}{dt}$$

$$i'(0) = c_1\left(\frac{4+\sqrt{2}}{2}\right) + c_2\left(\frac{4-\sqrt{2}}{2}\right) \quad c_1 = \frac{3+5\sqrt{2}}{2} \quad c_2 = \frac{-1-5\sqrt{2}}{2}$$

$$i(t) = \left(\frac{3+5\sqrt{2}}{2}\right) e^{\left(\frac{4+\sqrt{2}}{2}\right)t} + \left(\frac{-1-5\sqrt{2}}{2}\right) e^{\left(\frac{4+\sqrt{2}}{2}\right)t}$$

5pts

(3) $R = 1,\ C = 2,\ L = 1,\ V_c(0^-) = 1,\ I_L(0^-) = 1$

$$\frac{\partial^2 i(t)}{\partial t^2} + \frac{\partial i(t)}{\partial t} + \frac{1}{2} i(t) = 0 \quad \lambda_1 = \frac{1+i}{2} \quad \lambda_2 = \frac{1-i}{2}$$

$$i(t) = c_1 e^{-\frac{1}{2}t} \cdot \cos(t/2) + c_2 e^{-\frac{1}{2}t} \sin(t/2)$$

$$\left.\begin{array}{l} i(0) = 1 = c_1 \\ i'(0) = -2 \end{array}\right\} c_2 = -3$$

$$i(t) = e^{-\frac{1}{2}t} \cos(t/2) - 3e^{-\frac{1}{2}t} \sin(t/2)$$

5pts

Q3

(a) $h(t) = e^{j\omega_0 t}$

$x(t) = u(t+0.5) - u(t-0.5)$

$y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau)$ → só está definida p/ $-0.5 \leq \tau \leq 0.5$

$$y(t) = \int_{-0.5}^{+0.5} x(\tau)\, h(t-\tau)\, d\tau \rightarrow y(t) = \int_{-0.5}^{+0.5} e^{j\omega_0 (t-\tau)}\, d\tau \;,\; y(t) = e^{j\omega_0 t} \cdot \int_{-0.5}^{+0.5} e^{j\omega_0 \tau}\, d\tau$$

$$y(0) = 1 \cdot \int_{-0.5}^{+0.5} e^{j\omega_0 \tau}\, d\tau = \frac{2}{\omega_0} \operatorname{sen}\left(\omega_0/2\right)$$

PARA $\omega_0 = 2\pi$ ou $2\pi k$ p/ $k = 1, 2, 3, \dots$

1.5pts

(b) $y(t) = h(t) * x(t) = y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$

$h(t) = u(t+2) - u(t+1) + \delta(t-3)$

$$y(t) = \int_{-2}^{-1} x(t-\tau)\, d\tau + x(t-3)$$

$$y(0) = \int_{-2}^{-1} x(-\tau)\, d\tau + x(-3)$$

VALORES DE $x(t)$ NECESSÁRIOS

$$\boxed{\begin{array}{l} x(t),\ 1 \leq t \leq 2 \\ x(t),\ t = -3 \end{array}}$$

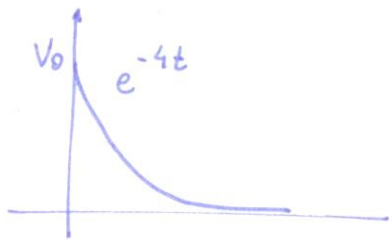
1.5pts

Q4

$$\frac{\partial y(t)}{\partial t} + 4\,y(t) = x(t)$$

RESPOSTA A ENTRADA ZERO:

$$\frac{\partial y}{\partial t} + 4\,y(t) = 0$$



$$\left.\begin{aligned} y(t) &= e^{\lambda t} \\ \frac{\partial y}{\partial t} &= \lambda e^{\lambda t} \end{aligned}\right\} \quad (\lambda + 4)\,e^{\lambda t} = 0 \quad \boxed{\lambda = -4}$$

COMO $V_0 = 0$ NÃO HÁ RESPOSTA ~~AO~~ ~~ESTA~~ ENTRADA ZERO.

RESPOSTA AO ESTADO ZERO:

SEMELHANTE AO CALCULO SUPOREMOS QUE $x(t) = e^{(-1+3j)t}$ e $y(t) = k\,e^{(-1+3j)t}$

$$(-1+3j)\,k\,e^{(-1+3j)t} + 4\,k\,e^{(-1+3j)t} = e^{(-1+3j)t} \qquad p/\ t > 0$$

$$(-1+3j)\,k + 4k = 1$$

$$-k + 3jk + 4k = 1$$

$$3(1+j)\,k = 1 \rightarrow \boxed{k = \frac{1}{3+j3}} \quad \text{ou} \quad k = \frac{\cancel{3} - j\cancel{3}}{\cancel{18}}$$

$$\boxed{k = \frac{1-j}{6}}$$

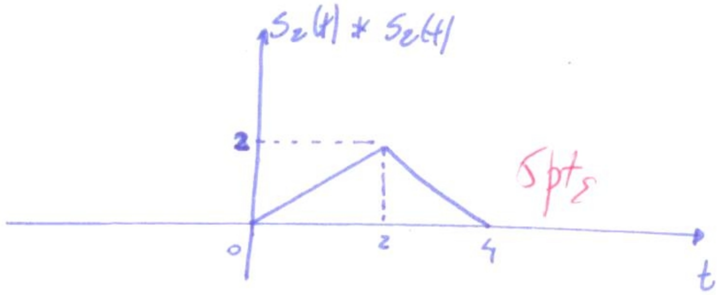
Lopes

Q5 -

1

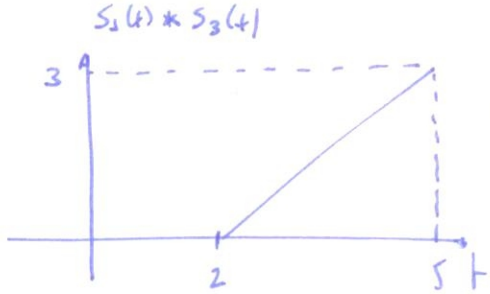
10

2



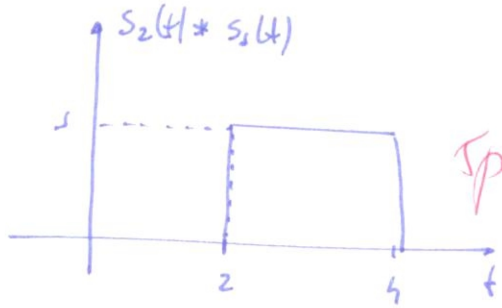
5pts

3



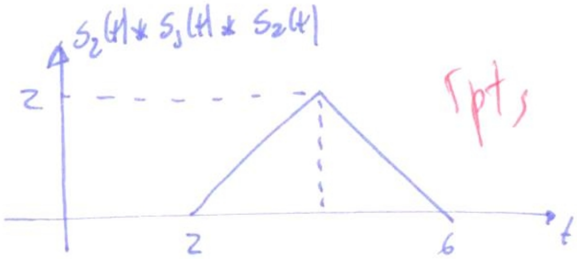
5pts

4



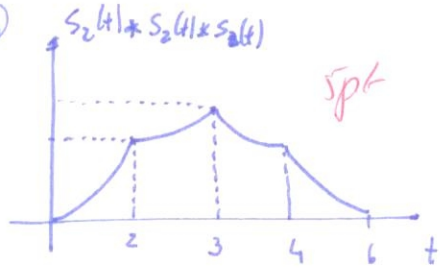
5pt

5



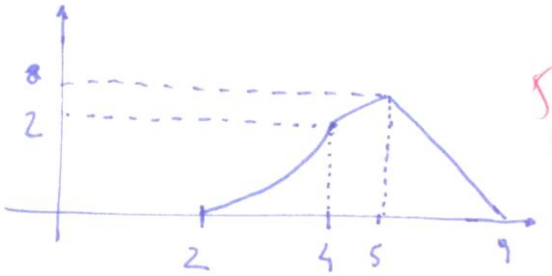
5pts

6



5pt

7



5pts