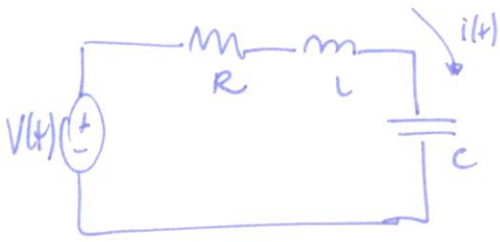
SISTEMAS LINEARES I

LISTA 2a

GABARITO

Q1

(a) MODELAGEM



$$\frac{\partial^2 i(t)}{\partial t^2} + \frac{R}{L}\frac{\partial i(t)}{\partial t} + \frac{1}{LC} i(t) = 0$$

$i(t) \xrightarrow{\mathcal{L}\{\cdot\}} I(s)$

$$i'(t) = sI(s) - i(0)$$
$$i''(t) = s^2 I(s) - s\,i(0) - i'(0)$$

$$s^2 I(s) - s\,i(0^-) - i'(0^-) + \frac{R}{L}\left[sI(s) - i(0^-)\right] + \frac{1}{LC} I(s) = 0$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) I(s) = \left(s + R/L\right) i(0^-) + i'(0^-)$$

Por LKT $\Rightarrow V(0) = i(0).R + V_c(0) + L\frac{di(t)}{dt} \rightarrow 0 = i(0).R + V_c(0) + L\,i'(0)$

$$i'(0) = \frac{-V_c(0) - i(0).R}{L}$$

(b) $R = 3,\ C = 2,\ L = 1,\ V_c(0^-) = 1 \ \text{e} \ I_L(0^-) = 1$

$$\left(s^2 + 3s + 1/2\right) I(s) = (s+3)\,i(0) + i'(0)$$

$$i'(0) = \frac{-3 - 1}{1} = \boxed{i'(0) = -4}$$

$$I(s) = \frac{s+1}{s^2 + 3s + 1/2} \rightarrow \frac{s+1}{\left(s + \frac{3-\sqrt{7}}{2}\right)\left(s + \frac{3+\sqrt{7}}{2}\right)} = \frac{\left(\frac{1}{2} + \frac{5\sqrt{7}}{14}\right)}{s + \frac{3-\sqrt{7}}{2}} + \frac{\left(\frac{1}{2} - \frac{5\sqrt{7}}{14}\right)}{s + \frac{3+\sqrt{7}}{2}}$$

$$i(t) = \left(\frac{1}{2} + \frac{5\sqrt{7}}{14}\right) e^{\left(\frac{3+\sqrt{7}}{2}\right)t} + \left(\frac{1}{2} - \frac{5\sqrt{7}}{14}\right) e^{\left(-\frac{3-\sqrt{7}}{2}\right)t}$$

Q1

(a)

2- $R=2,\ C=2,\ L=1,\ V_c(0^-)=1,\ I_L(0^-)=1$

$$\left(s^2+2s+\tfrac{1}{2}\right)I(s) = (s+2) + i'(0) \quad -3$$

$$I(s) = \frac{s+1}{s^2+2s+\tfrac{1}{2}} = \frac{s+1}{\left(s-\frac{-4-\sqrt{8}}{2}\right)\left(s-\frac{-4+\sqrt{8}}{2}\right)} = \frac{\frac{3+5\sqrt{2}}{2}}{s-\left(\frac{-4-\sqrt{8}}{2}\right)} + \frac{\frac{-1-5\sqrt{2}}{2}}{s-\frac{-4+\sqrt{8}}{2}}$$

$$i(t) = \frac{3+5\sqrt{2}}{2}\, e^{\left(\frac{-4+\sqrt{8}}{2}\right)t} + \left(\frac{1-5\sqrt{2}}{2}\right) e^{\left(\frac{-4-\sqrt{8}}{2}\right)t}$$

3- $R=1,\ C=2,\ L=1,\ V_c(0^-)=1,\ I_L(0^-)=1$

$$\left(s^2+1s+\tfrac{1}{2}\right)I(s) = (s+1)\,i(0^-) + i'(0) \quad -2$$

$$I(s) = \frac{s-1}{s^2+1s+\tfrac{1}{2}} = \frac{s-1}{\left(s-\frac{-1}{2}-\frac{i}{2}\right)\left(s-\frac{-1}{2}+\frac{i}{2}\right)} \rightarrow$$

$$i(t) =$$

Q2

(a)

$$\frac{\partial^3 y(t)}{\partial t^3} + 6\frac{\partial^2 y(t)}{\partial t^2} + 11\frac{\partial y(t)}{\partial t} + 6y(t) = x(t)$$

$x(t) \xrightarrow{\mathcal{L}\{\cdot\}} X(s)$

$y(t) \xrightarrow{\mathcal{L}\{\cdot\}} Y(s)$

$\frac{\partial y(t)}{\partial t} \xrightarrow{\mathcal{L}\{\cdot\}} sY(s) - y(0)$

$\frac{\partial^2 y(t)}{\partial t^2} \xrightarrow{\mathcal{L}\{\cdot\}} s^2Y(s) - sy(0) - y'(0)$

$\frac{\partial^3 y(t)}{\partial t^3} \xrightarrow{\mathcal{L}\{\cdot\}} s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$

$$(s^3 + 6s^2 + 11s + 6)Y(s) - s^2y(0) - sy'(0) - y''(0) - 6sy(0) - 6y'(0) - 11y(t) = X(s)$$

$$(s^3 + 6s^2 + 11s + 6)Y(s) = X(s) + (s^2 + 6s + 11)y(0) + (s + 6)y'(0) + y''(0)$$

$(s+1)(s+2)(s+3)$

P/ ESTADO ZERO → $y''(0) = 0$, $y'(0) = 0$, $y(0) = 0$

$x(t) = e^{-4t}u(t) \xrightarrow{\mathcal{L}\{\cdot\}} X(s) = \frac{1}{s+4}$

$$Y(s) = \frac{1}{(s+1)(s+2)(s+3)} \cdot \frac{1}{(s+4)} \Rightarrow Y(s) = \frac{1}{6}\frac{1}{s+1} + \frac{-1/2}{s+2} + \frac{1/2}{s+3} + \frac{-1/6}{s+4}$$

$$y(t) = \frac{1}{6}e^{-t} - \frac{e^{-2t}}{2} + \frac{1}{2}e^{-3t} - \frac{1}{6}e^{-4t}$$

(b) P/ ENTRADA ZERO $X(s) = 0$, $y(0^-) = 1$, $y'(0) = -1$, $y''(0) = 1$

$$Y(s) = \frac{1}{(s+1)(s+2)(s+3)} \cdot (s^2 + 6s + 11 - s - 6 + 1)$$

$$Y(s) = \frac{1}{(s+1)(s+2)(s+3)} \cdot (s^2 + 5s + 6) \rightarrow Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+2)(s+3)} = Y(s) = \frac{1}{s+1}$$

$$y(t) = e^{-t}$$